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Free Vibration Analysis of Orthotropic Thin Rectangular SSSS Plate Using Polynomial Series Function

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Abstract

Free vibration analysis of material orthotropic rectangular thin plate simply supported on all edges (SSSS) plate using Taylor's series function in Ritz method was carried out. A Taylor's series truncated at the 5th terms was used to develop the polynomial shape function which satisfied the boundary condition of the plate. This was substituted into the potential energy functional of Ritz and minimized the resulting functional to get the free vibration equation. Three sets of the flexural rigidity ratios for x, y and xy were used. The values of the flexural rigidities for the three sets are 1.0; 1.0; 1.0 and 1.0; 0.5; 1.0 and 1.0; 0.5; 0.5 respectively. The fundamental frequencies were obtained for various aspect ratio ranging from 0.1 to 2 at increments of 0.1. Fundamental natural frequency of the plate for aspect ratio of 1.0 for the three sets of flexural rigidity ratios are respectively 19.75, 17.10 and 15.61. The corresponding values from the works of Pilkey are respectively 19.74, 17.10 and 15.61. The closeness between the value from the present study and those of Pilkey indicates that the approach of the present study is satisfactory and reliable.

Keywords: Ritz, orthotropic plate, fundamental frequency, Taylor-Series, thin plate, potential energy functional.

NOTATION

$\varphi_1 = \varphi_2 = \varphi_3 = \frac{D}{D_{i(i=x, xy \text{ or } y)}} = \text{flexural rigidity ratios in } x, x - y \text{ and } y \text{ respectively.}$

W = Deflection function

$W^{IR} = \frac{\partial w(R, Q)}{\partial R} = \text{The first partial derivative of deflection with respect to } R$

$W^{IIR} = \frac{\partial^2 w(R, Q)}{\partial R^2} = \text{The second partial derivative of deflection with respect to } R$

$W^{IQ} = \frac{\partial w(R, Q)}{\partial Q} = \text{The first partial derivative of deflection with respect to } Q$

$W^{IIQ} = \frac{\partial^2 w(R, Q)}{\partial Q^2} = \text{The second partial derivative of deflection with respect to } Q$

$W^{IIRQ} = \frac{\partial^2 w(R, Q)}{\partial R \partial Q} = \text{The second partial derivative of deflection with respect to } R \text{ and } Q$

$D_x = \frac{E_x t^3}{12(1 - \mu_x \mu_y)} = \text{flexural rigidity in } x \text{ direction}$

$D_y = \frac{E_y t^3}{12(1 - \mu_x \mu_y)} = \text{flexural rigidity in } y \text{ direction}$

E_x, E_y are the young modulus in the x and y directions respectively.

μ_x, μ_y and t are the Poisson ratios in the x and y directions and thickness of the plate respectively.

Introduction

Different theories have been introduced to handle the vibration of plate problems. Correspondingly, many powerful new methods have also been developed to analyze these problems.

(Srinivas et al. 1970) examined exact three-dimensional plate theory to study the vibration of simply supported homogeneous and laminated thick rectangular plate. Direct integration solutions to fully

clamped plate have for centuries not been obtained and therefore it is currently considered that an exact solution is not achievable for rectangular problem of this type. Direct integration method is only limited to analyzing and finding the exact solutions of simply supported plates on all four edges that is the SSSS plate. In addition if one of the edges is not simply supported, it becomes difficult to work with thereby justifying the need for other approaches. The numerical approach has wider application to quite a number of cases and boundary conditions which cannot be easily undertaken using the equilibrium (Euler approach), and its solution approaches closely those of the exact approach (Ventsel and Kranthammer, 2001). The Ritz approximate approach which is a generalized version of the Raleigh method (Raleigh, 1877) can also be applied in the solution of the SSSS plate. The use of Trigonometric series in formulating the shape functions of a plate whose edges are simply supported (SSSS) or clamped (CCCC) and a plate whose opposite edges are clamped and other opposite edges simply supported is possible. It cannot be employed when opposite edges are simply supported and clamped. The shape functions of SCCC, CSSS, SSCF, CSSF, plates cannot be formulated using trigonometric series. Many other boundary conditions limit the use of trigonometric series (Ugural 1999, 1998; Ventsel and Krauthammer, 2001). Therefore, in a bid to overcome the deficiencies of the Trigonometric series, and to arrive at very close approximations to the exact solutions with reduced computational effort and volume of work, Taylor-Maclaurin Series will be used in formulating the polynomial shape functions. Its obvious advantages lie in the ease of application and versatility to different plate cases with different boundary conditions which hitherto was a problem with the trigonometric series. It is noteworthy that no previous researcher has applied this in rectangular orthotropic plate. Ibearugbulam (2012) used truncated Taylor-Mclaurin's series as shape function in Ritz method but it was only applied in the isotropic case. In view of this, the research study on free vibration analysis of material orthotropic plate using Taylor's series aims to fill that gap.

Mathematical Expression For Total Potential Energy Functional For Vibrating Material Orthotropic Plate

Abamara (2014) using a deflection function technique based on Ibearugbulam et al. (2013), derived the equation for the fundamental frequency of

the vibrating continuum. This was achieved by employing the principle of conservation of energy where the strain and kinetic energies of the continuum were derived from the first principles using the theory of elasticity and this is given as

$$\pi = \frac{D_x}{2b^2} \int_0^1 \int_0^1 \left[\frac{\phi_1}{p^3} \left(\frac{\partial^2 w}{\partial R^2} \right)^2 + 2 \frac{\phi_2}{p} \left(\frac{\partial^2 w}{\partial R \partial Q} \right)^2 + p \phi_3 \left(\frac{\partial^2 w}{\partial Q^2} \right)^2 \right] \partial R \partial Q - \frac{pb^2 \lambda^2 \rho t}{2} \int_0^1 \int_0^1 w^2 \partial R \partial Q \quad (1)$$

Truncated Taylor-maclaurin series

Ibearugbulam (2012) expanded the general shape function using the Taylor-Maclaurin Series and obtained

$$w = \sum_{m=0}^4 \sum_{n=0}^4 I_m J_n x^m \cdot y^n \quad (2)$$

Transforming the x-y coordinate system to R-Q coordinate system, where

$$R = \frac{x}{a}; Q = \frac{y}{b}. \text{ Since } x = aR \text{ and } y = bQ \text{ and letting } a_m = I_m \cdot a^m, \text{ and } b_n = J_n \cdot b^n$$

We have that:

$$w = \sum_{m=0}^4 \sum_{n=0}^4 a_m b_n R^m Q^n \quad (3)$$

The function in equation (3) can be further expanded in the form

$$w(R, Q) = (a_0 + a_1 R + a_2 R^2 + a_3 R^3 + a_4 R^4)(b_0 + b_1 Q + b_2 Q^2 + b_3 Q^3 + b_4 Q^4) \quad (4)$$

a and b are the plate dimensions and R and Q represent the dimensionless axes parameters

Boundary Conditions And Deflection Function For Ssss Plate

The boundary conditions for an orthotropic SSSS plate are

$$W(R=0) = W^{II}R(R=0) = 0; W(R=1) = W^{II}R(R=1) = 0 \quad (5)$$

$$W(Q=0) = W^{II}Q(Q=0) = 0; W(Q=1) = W^{II}Q(Q=1) = 0 \quad (6)$$

Applying the boundary conditions of equations (5) and (6) in equation (4) gives

$$W = A(R - 2R^2 + R^4)(Q - 2Q^3 + Q^4) \quad (7)$$

And $W = AH$, where A is the amplitude of the deflected shape and H is the shape of the deflected curve.

Application of rayleigh- ritz method

Let the partial derivatives of equation (7) be expressed as

$$W^{IRR} = A(12R^2 - 12R)(Q - 2Q^3 + Q^4) \quad (8)$$

$$W^{IIQ} = A(R^2 - 2R^2 + R^4)(12Q^2 - 12Q) \quad (9)$$

$$W^{IIRQ} = A(1 - 6R^2 + 4R^3)(1 - 6Q^2 + 4Q^3) \quad (10)$$

Integrating the square of equations (7) (8), (9), (10) partially with respect to R and Q , will give respectively:

$$\int_0^1 \int_0^1 (W)^2 = A^2 (0.04920635)(0.04920635) = 0.00242127A^2 \quad (11)$$

$$\int_0^1 \int_0^1 (W^{IRR})^2 \partial R \partial Q = A^2 (4.8)(0.04921) = 0.23621A^2 \quad (12)$$

$$\int_0^1 \int_0^1 (W^{IIQ})^2 \partial R \partial Q = A^2 (0.04921)(4.8) = 0.23621A^2 \quad (13)$$

$$\int_0^1 \int_0^1 (W^{IIRQ})^2 \partial R \partial Q = A^2 (0.48571)(0.48571) = 0.23591A^2 \quad (14)$$

We substituted equations (11), (12), (13) and (14) into equation (1), carried out the integration, minimize the resulting functional and made the fundamental frequency the subject of the equation to obtain:

$$\lambda^2 = \frac{D_x}{a^4 p t} \left[\varphi_1 \cdot 97.5562 + \frac{\varphi_2}{p^2} \cdot 194.8647 + \frac{\varphi_3}{p^4} \cdot 97.5562 \right] \quad (15)$$

Where $p = b/a$ is the aspect ratio

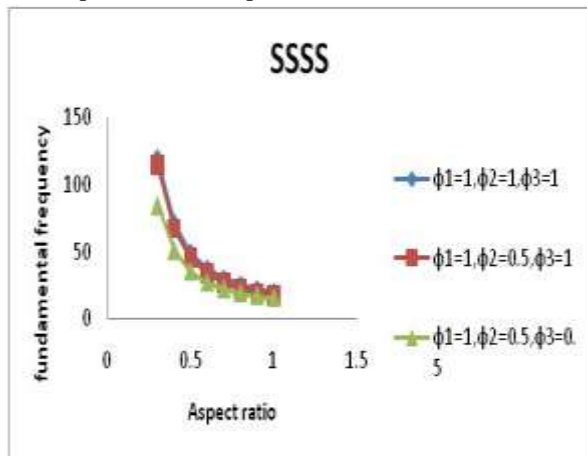


Figure 1.1: Fundamental frequency λ against the aspect ratios

Results and discussion

The values of the natural frequency λ for the different aspect ratios, from the present study and the work of Pilkey for the different combinations of $\varphi_1, \varphi_2, \varphi_3$ are shown in tables A1-A3 in appendix A. The graph on figure 1.1 was plotted from tables A1-A3 for the fundamental frequency obtained from present study against the aspect ratios from 0.3 to 1.0. From figure 1.1, it can be deduced that for the individual graphs, the fundamental frequency decreased with increased aspect ratio for the different combinations of flexural rigidity ratios and the values of the frequency decreased with decreasing flexural rigidity ratios of ϕ_2 and ϕ_3 . Similarly, from tables A1-A3, the average percentage difference between the values from Pilkey (2005) and the present study are respectively 0.0458%, 0.0564%, 0.0528%, for the three different cases of flexural rigidity ratios studied herein.

Table A1 (SSSS plate)

$$\lambda^2 = \frac{D_x}{a^4 p t} \left[\varphi_1 \cdot 97.5562 + \frac{\varphi_2}{p^2} \cdot 194.8647 + \frac{\varphi_3}{p^4} \cdot 97.5562 \right]$$

Frequency parameter λ for $\varphi_1 = \varphi_2 = \varphi_3 = 1, p = \frac{b}{a}$

P	λ^2	λ (Present)	λ (Pilkey)
0.1	995146	997.570	996.833
0.2	65941.8	256.791	256.613
0.3	14306.69	119.611	119.535
0.4	5126.25	71.598	71.558
0.5	2437.914	49.375	49.351
0.6	1391.595	37.304	37.288
0.7	901.5541	30.026	30.014
0.8	640.2066	25.302	25.293
0.9	486.821	22.064	22.056
1	389.9771	19.748	19.741

Table A2 (SSSS plate)

$$\lambda^2 = \frac{D_x}{a^4 p t} \left[\varphi_1 \cdot 97.5562 + \frac{\varphi_2}{p^2} \cdot 194.8647 + \frac{\varphi_3}{p^4} \cdot 97.5562 \right] \quad \text{For } p = \frac{b}{a}$$

Frequency parameter λ for $\varphi_1 = 1, \varphi_2 = 0.5, \varphi_3 = 1, p = \frac{b}{a}$

P	λ^2	λ (Present)	λ (Pilkey)
0.1	985402.8	992.675	991.934
0.2	63505.99	252.004	251.822
0.3	13224.11	114.996	114.917
0.4	4517.297	67.211	67.168
0.5	2048.185	45.257	45.230

0.6	1120.95	33.481	33.462
0.7	702.7126	26.509	26.495
0.8	487.9686	22.090	22.079
0.9	366.5341	19.145	19.136
1	292.5448	17.104	17.096

Table A3 (SSSS plate)

$$\lambda^2 = \frac{D_x}{a^4 \rho t} \left[\varphi_1 \cdot 97.5562 + \frac{\varphi_2}{p^2} \cdot 194.8647 + \frac{\varphi_3}{p^4} \cdot 97.5562 \right]$$

Frequency parameter λ for $\varphi_1 = 1, \varphi_2 = 0.5, \varphi_3 = 0.5, p = \frac{b}{a}$

P	λ_2	λ (Present)	λ (Pilkey)
0.1	497621.8	705.423	704.902
0.2	33019.68	181.713	181.587
0.3	7202.126	84.865	84.812
0.4	2611.903	51.107	51.078
0.5	1267.735	35.605	35.587
0.6	744.5758	27.287	27.275
0.7	499.5552	22.351	22.341
0.8	368.8814	19.206	19.198
0.9	292.1886	17.094	17.086
1	243.7667	15.613	15.606

Conclusion

The results show that the solutions of the SSSS thin rectangular orthotropic plate by use of polynomial deflection function are very close to the values of earlier analysis by use of other deflection function. The effectiveness of the polynomial function have been validated and it can be applied in all boundary conditions. The fundamental frequencies of plates with any combination of boundary conditions can now be obtained using this present approach.

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